A MODEL OF PRICING ON A TWO-SIDED MATCHING PLATFORM WITH HORIZONTALLY DIFFERENTIATED AGENTS*

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Abstract

This paper presents preliminary results from a research project on externalities and pricing and two-sided matching platforms. I study how a monopolist firm operating a two-sided matching platform optimally sets its prices in the presence of agents who are horizontally-differentiated, á la Salop [1979]. Agents are searching for their ideal match (their own type) and face a two-part tariff pricing schedule set by the firm. In the steady-state equilibrium, profit maximization dictates that two-part tariffs reduce to a uniform pricing – that is, the monopolist optimally sets per-transaction prices to 0 and charges all agents the same fixed fee to join the platform regardless of their type. The results of the paper may be interpreted in terms of matching externalities – when there are none, the resulting optimal pricing strategy is non-distortionary. I conclude the paper by presenting more complicated extensions that can be studied from this model.

JEL Classification: D21, C78, D83.

Keywords: Pricing, Monopolist, Matching, Search, Two-Sided Platforms

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1 Introduction

The study of two-sided matching markets is relatively new in industrial organization. In these types of markets, a *platform* functions as an intermediary to bring economic agents together. These platforms generally cater to two or more "sides" who would otherwise have trouble finding each other. In this paper, I present a simple theoretical model of pricing on a two-sided matching platform with search. For ease of exposition, I focus the discussion of the paper on heterosexual online dating websites, where the two sides are men and women. The online dating industry is extremely successful, with total revenues from 2012 reaching $2.1 billion.[1] One of the biggest players in this market, eHarmony, reported that they were responsible for nearly 5% of marriages in the US in the year 2009.[2]

A dating platform's ability to charge a fee for its service depends on a variety of factors. Rochet and Tirole [2006] argue that a platform's value, and hence the fee it can charge, crucially depends on the benefits or access it is able to provide to its users. If it is very difficult for economic agents on one side of the platform to get access to members of the opposite side, they may be willing to pay more for the services offered by a platform. Agents may also be willing to pay more to a platform that provides access to high quality potential dates. It is also important to note that prices set by the platforms may be different for both sides. For example, a platform that is trying to attract more women may waive the fees for women but not for men. Furthermore, externalities (the uncompensated benefits or costs imposed on agents that use a platform) play a significant role in optimal pricing. Armstrong [2006] stresses the importance of the relative sizes of the externalities imposed across the two sides. For concreteness, suppose that the two sides of the market are sides A and B. If A exerts a large positive externality on B, the platform could aggressively target agents on side A, charging them low, zero, or even negative (subsidies) prices. In practice, online dating platforms utilize a variety of pricing structures. Many charge term payments where a fixed amount of money is paid on a regular basis. For example, a platform may charge $30/month for a 12 months and offer small discounts for contracts that extend beyond 12 months. Some platforms charge a more general pricing structure called a two-part tariff, which consists of a fixed fee for a membership and per-transaction fees for each interaction they have with members on the platform, and others charge uniform one-time fee to join a platform to all participants.

In this paper, agents have preferences distributed on a unit circle, as described by Salop [1979]. Since agents are horizontally differentiated, each individual on the platform has different preferences over who their ideal match is. In this model, economic agents exhibit preferences for similarity – a person's ideal match is their own type. This is in contrast to matching with vertical differentiation where all agents agree on the ranking of types compete for the partner with the highest quality. Utility is non-transferable – agents may not bargain over the surplus that results from a match. The platform in this paper is a monopolist and charges a two-part tariff. I assume that types are perfectly observable to the monopolist. The main result of the paper is that two-part tariff pricing reduces to uniform pricing in a steady-state equilibrium and per-transaction prices are set to zero. An intuitive explanation for the pricing result is as follows: in a world where everyone is self-interestedly pursuing their ideal (own) type, there are no matching externalities since an individual's ability to obtain her ideal type does not take away from other agents' abilities to obtain their ideal types. With observable types, the first-best outcome is achieved. In particular, there is full surplus extraction and the monopolist acts as a social planner by choosing fixed and per-transaction fees to maximizes total surplus Since there are no externalities in the market, the monopolist set per-transaction prices to be zero. This price of zero does not distort the agents' matching decisions and can be likened to a perfectly competitive outcome in a market with no externalities, where agents are behaving optimally from both their own point of view and the social planner's. In the concluding section of this paper, I present a more rich environment that can be studied as a result of this simple model.

[1] IBISWorld Dating Services 2014 Market Research Report.
[2] www.eharmony.com

2 Literature Review

This paper is one of the few that studies externalities and how they affect optimal pricing on two-sided matching platforms with search and non-transferable utility. Recent work by Halaburda and Piskorski [2011] and Damiano and Li [2007] have explored similar environments but with different focuses. Halaburda and Piskorski [2011] explored competition among search platforms and found that increasing the number of agents on each side of the market not only has a positive effect due to large choice but also a negative effect due to competition between agents on the same side. Agents resolve the trade off between these externalities differently – agents with low outside options face stronger competition effects than they do choice effects. Thus, these agents have a higher willingness to pay for a platform restricting choice, and vice versa. Damiano and Li [2007] study the problem of a monopoly matchmaker that uses a schedule of entrance fees to sort different types of agents on the two sides of a matching market into exclusive 'meeting places', where agents randomly form pairwise matches and obtain a match with probability one.

Weyl [2010] studies heterogeneity on two-sided platforms by developing a theory of monopolistic platform pricing. In his paper, Weyl extends the basic framework of Rochet and Tirole [2006] and propose a more plausible (yet equally tractable) model of heterogeneity in which agents differ in their income. Weyl's model incorporates a continuum of users and gives a general measure of market power in order to study policy questions, including price regulation and dealing with platform mergers. Hagiu [2006] studies two-sided platforms in which the sellers and buyers on the platform do not arrive at the same time. Cabral [2011] considers a dynamic model of competition between networks where consumers die and firm continuously compete Bertrand-style for new consumers. Hoppe et al. [2011] integrate costly signaling with heterogenous agents into a basic assortative matching framework. Kojima and Pathak [2009] study stability in two-sided matching markets. They find that under some regularity conditions, the proportion of agents with incentives to lie about their preferences when all other agents are truthful approached zero as the market becomes very large. Azevedo and Leshno [2012] found that in both discrete and continuum two-sided matching markets with heterogeneous agents, stable matchings have a very simple structure with matches being formed if agents on each side are ranked above a certain threshold, a result similar to Burdett and Coles [1997]. In this paper, I also obtain matching thresholds for individuals on the platform. This being said, unlike the previous two papers mentioned, my thresholds are also a function of prices charged by the platform.

3 Model

3.1 Environment

This is an infinite-horizon model with discrete time. There is a single monopolist operating a matching platform that caters to two sides. I call one side 'women' and the other 'men'. Each side of the market has characteristics distributed on the circumference of the unit circle. Denote this characteristic by $x \in [0, 1]$. Let x be distributed according to the uniform distribution with cumulative function G and the corresponding density function g. Agents and the firm discount at the same rate δ. The monopolist platform charges a two-part tariff to all agents – agents get charged a fixed fee f to first enter the platform and then get charged a fee p for each interaction they have with others on the platform. I will be focusing on a steady-state equilibrium.

Faced with the pricing schedule set by the monopolist, timing in this model proceeds as follows:

1. Faced with f and p, agents decide whether or not to join the platform.
2. Agents who decide to join the platform pay f and receive a random draw of a new match after paying the per-transaction fee p.
3. Agents meet up, learn each other's type, and decide whether or not to marry. If both agents decide to marry, they obtain their matching utility (defined below) and leave the platform forever. If one or

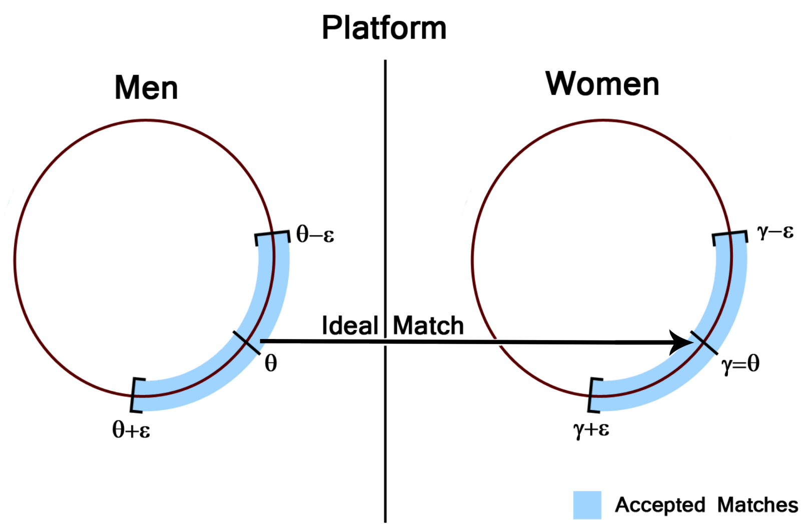

Figure 1: Illustration of the threshold $\epsilon-$strategy employed by agents

both of the agents decide not to marry, they continue searching by proceeding to the next period by paying another p for another draw.

I assume that there is a matching benefit $b > 0$ which is common to all agents. This term captures the overall benefit that individuals obtain when married. An agent's *ideal type* is assumed to be her own and agents face a cost which increases in distance with marrying a type that is not her own. This cost is denoted by $c > 0$ and serves as a per-unit distance cost of marrying away from an agent's ideal type.

I can now define the matching utility obtained by agent x when he chooses to marry agent y as

$$u^x(y) \equiv \frac{b - c \mid x - y \mid}{1 - \delta} \tag{1}$$

This utility is multiplied by $\frac{1}{1-\delta}$, which is a result of geometric discounting and takes into account the fact that a marriage lasts forever if agents decide to leave the platform. The distance metric $\mid$ measures the shortest arc-length between x and y on the unit circle.

3.2 Agent's Problem

In general, agents can choose to marry whomever they want. This being said, optimality will dictate that all agents employ a 'threshold' strategy when deciding who to marry. All agents obtain 0 as a reservation utility.

Definition 1 *An agent's strategy is a function* $s : [0, 1] \to \{accept,\ reject\}$

Faced with the draw of a match in a particular period, an agent's strategy provides an accept or reject answer to the match. A marriage only forms if and only if both agents accept; in the case where one or both agents reject the match, agents continue searching and proceed to the next period. In steady-state, I can define the Bellman Equation for the agents. The Bellman Equation does not involve f since agents treat this cost as sunk when they are making their marrying decisions once on the platform.

Taking into account the fact that the per-transaction price p must be paid for a new random draw, a type x's Bellman equation is given by

$$V_x(.) = \max\{\frac{u^x(y)}{1-\delta}\,,\ \delta(\int_0^1 V_x(.)\,dG(.) - p)\} \tag{2}$$

Using standard techniques used in search models, I show that the optimal strategy for the agent will be a threshold strategy, where agent x will accept and marry all agents who are in the ϵ-ball around x; that is, x will accept any agent in $[x-\epsilon, x+\epsilon]$ and reject all agents outside of this interval. To see this, note that the right-hand side is the expected continuation utility of rejecting their current draw and continuing the search procss. This value, which is denoted C_x, is constant regardless of the draw. The left-hand side of what the agents obtain if they accept their current draw and it increases as $\mid x-y \mid$ decreases, where y is the type of the agent that x is deciding whether or not to marry. Next, I derive ϵ. For a visual aid on the ϵ-threshold strategy, please refer to Figure 1.

Proposition 1 *All agents have the same ϵ-threshold given by*

$$\epsilon = \min\{\frac{1}{2}, \frac{c(\delta-1)+\sqrt{c(\delta-1)(c(\delta-1)-4\delta(b+p\delta))}}{2c\delta}\}$$

Note that the ϵ depends on the per-transaction price p. For ease of notation, define

$$\bar{\epsilon} = \frac{c(\delta-1)+\sqrt{c(\delta-1)(c(\delta-1)-4\delta(b+p\delta))}}{2c\delta}$$

The expression for ϵ includes the min function to ensure that the ϵ-interval does not wrap twice around the circle. I can now demonstrate simple comparative statics on the ϵ-thresholds.

Proposition 2 *In the case where $\bar{\epsilon} \leq \frac{1}{2}$, we have that*

- $\frac{\partial \epsilon}{\partial p} \geq 0$
- $\frac{\partial \epsilon}{\partial b} \geq 0$
- $\frac{\partial \epsilon}{\partial \delta} \leq 0$
- $\frac{\partial \epsilon}{\partial c} \leq 0$

To understand these results, let's frame the ϵ-ball in terms of "pickiness" for choosing a marriage partner. A large ϵ indicates that an agent is less picky because she is willing to accept a wide range of types that are not her own. Similarly, a small ϵ indicates that an agent is more picky about who she marries. Unsurprisingly, the more patient individuals are, they less picky they become in each period. As $\delta \to 1$, the agent values the future more and is comfortable with staying on the platform to obtain a good marriage partner. As the matching cost c increases, agents face higher costs to marrying a type further away from their own and thus become more picky with regards to who they marry. As the matching benefit b increases, agents become less picky as getting married (to anyone) becomes more attractive. Proposition 2 also gives that an agent becomes less picky in each period as p increases. This is intuitive – a higher p corresponds to a higher cost of staying on the platform. This means that the platform has an imperfect mechanism to control the matching rates since increasing the per-transaction price cause staying on the platform to become more costly, encouraging agents to become less picky and marry at a faster rate. This mechanism of getting people to marry faster is imperfect since it only works up till a point. As p increases, ϵ will eventually be $\frac{1}{2}$, which means every individual on the platform will marry the first person they meet.

3.3 Dynamics

At the beginning of each period t, there is a mass μ_t of agents. Agents find matches and leave the platform at rate $o_t(p)$, and there are $i_t(f,p)$ agents who join the platform. Note that the outflow rate depends on the per-transaction price p, and since the firm gets to decide how many people to join the platform, the inflow rate is also an indirect choice variable from the firm. The firm essentially controls the inflow rate by changing the f. The transitional equation for μ_t is

$$\mu_{t+1} = (1 - o_t(p))\mu_t + i_t(f, p) \tag{3}$$

In steady-state, I am able to use the condition that total outflow from the platform, which is $o(p)\,\mu$, is equal to total inflow onto the platform, which is $i(f, p)$. This then allows me to solve for a constant mass of agents on the platform.

$$o(p)\,\mu = i(f, p) \tag{4}$$

I can simplify this further by recalling that agents find matches at rate $2\epsilon(p)$, giving that $o(p) \equiv 2\epsilon(p)$. The steady-state condition finally becomes

$$\mu = \frac{i(f, p)}{o(p)} \tag{5}$$

3.4 Monopolist's Problem

The monopolist steady-state profit can be written as

$$\Pi_{\text{ss}} = 2(i(f)f + \sum_{t=0}^{\infty} \delta^{t+1} p i(f)(1 - o(p))^t) \tag{6}$$

The monopolist can extract f from all of the agents who are allowed onto the platform and change then charge p for each period after. For ease of exposition, take an example. An agent who has been on the platform in period $t = 3$ would have been unsuccessful in finding a match in the past two time periods. This occurs with probability $(1-o)^2$. In particular, after T periods, there will be $i(1-o)^T$ agents on the platform. The monopolist can extract p from each of these $i(1-o)^T$ agents on the platform and discounts each of these periods by δ. This gives the expression $\sum_{t=0}^{\infty} \delta^{t+1} p i(f)(1 - o(p))^t$ in the profit function. The new cohort of agents who join the platform will be charged f and there are $i(f)$ of them, giving the term $i(f)\,f$. The constant in front of the profit function, although irrelevant to the optimization problem, is present to signify that the platform extracts this sum of profits from two sides of the market.

Evaluating (6) gives

$$\Pi_{\text{ss}} = 2(if + \frac{\delta i p}{1 - \delta(1 - o)}) \tag{7}$$

Since types are observable, we have only a participation constraint in the firm's optimization problem, which is given as follows.

$$\max_{p,f} \Pi_{\text{ss}}$$
$$\text{s. t.} \quad (IR_x)\; C_x(p) - f \geq 0 \;\forall x$$

where C_x is the expected lifetime utility of x being on the platform. By Proposition 1, the ϵ-thresholds for all agents are the same and so $C_x = C\ \forall x$. This means that the optimal f will be

$$C = f \tag{8}$$

Referring back to the dynamics, note that since the monopolist can observe types, he will charge f to bind the individual rationality constraint. As we will see later, the firm will also let in *all* agents whose individual rationality constraints bind onto the platform.

3.5 Equilibrium

Now, I solve for equilibrium by combining the steady-state conditions, firm optimality conditions, and agent's optimal matching behavior.

Proposition 3 *Consider the profit function given by* (7).

- Π_{ss} *is linear in* i *and* $\frac{\partial \Pi_{ss}}{\partial i} > 0$*, implying that* i *is optimally set to be* 1.
- *The firm optimally sets* $p = 0$.

From the proposition, the firm wants to let everyone who wants to join the platform on so that it can extract more surplus. However, combining the steady-state conditions with firm's optimality gives us the following proposition.

Proposition 4 *A symmetric, steady-state equilibrium in this model has the following characteristics:*

1. *The per-transaction price is* $p = 0$ *for all agents.*
2. *The fixed fee is then set so that* $f = C \mid_{p=0}$ *for all agents.*
3. i *is set so that*
$$i = 1$$
4. *The probability that any agent on the platform receives a match approaches* 1 *as time approaches* ∞.

The equilibrium characterization is very clean for two reasons. The first is that since agents are uniformly distributed over the circumference of a circle and the firm can perfectly observe types, agents are essentially identical to the firm. Secondly, given the utility specification, the ϵ-thresholds are the same for all agents, and that if one agent is willing to marry another, then a marriage necessarily forms. In other words, if one man's type is in a woman's ϵ-ball, then the woman's type is necessarily within the man's ϵ-ball. If the ϵ-thresholds were different in any way, the matching equilibrium would be very messy, leading to a correspondingly messy pricing equilibrium in steady-state.

4 Conclusion

This paper is one of the first that studies the effects of externalities on optimal pricing in markets operated by a two-sided platform. I presented a stylized model where a monopolist operates a two-sided matching platform in the presence of horizontally-differentiated agents. I then showed that in the steady-state equilibrium, the optimal pricing strategy is to charge the same fixed fee to all participants when they first join, regardless of type, and then charge nothing afterwards for further interactions with other participants on the platform. This paper is very simple but is a first step and baseline model for further research in this field. In particular, it highlights the need to study two understudied topics. The first, which requires a more involved model, is to consider modeling a market where individuals care about more than one characteristic when deciding who to marry. In a working paper, Nungsari and Flanders [2017] study this question by considering agents who care about multiple characteristics when matching. They find that when explicitly modeling this multidimensional aspect of matching, interesting externalities result that would otherwise be suppressed. For example, the platform may find it optimal to charge distortionary (i.e. negative or positive) per-interaction prices, depending on the parameters of the model. The second is the need to model competition amongst two-sided platforms, which can be easily studied using the same model and setup presented in this paper.

5 Appendix

Proposition (1)

Proof. The indifference condition for all agents is the following equation

$$\frac{u^x(\bar{y})}{(1-\delta)} = \delta(\int_0^1 V(y)\, dG(y) - \bar{p})$$

We may then repeat the steps done in Proposition 2 to obtain the ϵ. In particular, we note that since

$$\epsilon = \min\{\frac{1}{2}, \frac{c(\delta-1) + \sqrt{c(\delta-1)(c(\delta-1) - 4\delta(b+\bar{p}\delta))}}{2c\delta}\}$$

we have that $\epsilon \geq 0 \iff \bar{p} \geq \frac{-\alpha}{\delta}$ and $\bar{p}$ is a real number $\iff (\pi(\delta-1) - 2\delta(\alpha+\bar{p}\delta)) \leq 0 \iff \bar{p} \geq \frac{\delta(\pi-2\alpha)-\pi}{2\delta^2}$. Since $\frac{-\alpha}{\delta} \geq \frac{\delta(\pi-2\alpha)-\pi}{2\delta^2}$, we have that the condition that $\bar{p} \geq \frac{-\alpha}{\delta}$ ensures that $\epsilon \in \mathbb{R}_+$. ■

Proposition (2)

Proof. Taking derivatives of ϵ gives

$$\frac{\partial \epsilon}{\partial p} = \frac{\delta(1-\delta)}{\sqrt{c(\delta-1)(c(\delta-1) - 4\delta(b+p\delta))}} \geq 0$$

$$\frac{\partial \epsilon}{\partial b} = \frac{1-\delta}{\sqrt{c(\delta-1)(c(\delta-1) - 4\delta(b+p\delta))}} \geq 0$$

$$\frac{\partial \epsilon}{\partial \delta} = \frac{c(\delta-1) - 2\delta(b+p\delta^2) + \sqrt{c(\delta-1)(c(\delta-1) - 4\delta(b+p\delta))}}{2\delta^2\sqrt{c(\delta-1)(c(\delta-1) - 4\delta(b+p\delta))}} \leq 0$$

$$\frac{\partial \epsilon}{\partial c} = \frac{-(1-\delta)(b+p\delta)}{\sqrt{c(\delta-1)(c(\delta-1) - 4\delta(b+p\delta))}} \leq 0$$

■

Proposition (3)

Proof. It is easy to see from (7) that Π_{ss} is linear in i, and so i would optimally be set to be 1. To see the second point, refer to the following intuitive argument:

For ease of exposition, given an agent's matching threshold ϵ, write the expected discounted utility of agents as

$$EU(\epsilon) \equiv U - f - p\, D(\epsilon)$$

where U is the matching benefit, D is the expected number of dates the agent goes on before leaving the platform, p is the per-transaction price charged by the platform, and f is the fixed price. Recognizing that per-transaction prices affect the threshold ϵ, we take first-order conditions with respect to ϵ to obtain

$$\frac{\partial EU}{\partial \epsilon} = 0 \implies \frac{\partial U}{\partial \epsilon} = p\frac{\partial D(\epsilon)}{\partial \epsilon} \implies p = 0 \text{ since } \frac{\partial U}{\partial \epsilon} = 0$$

■

Proposition (4)

Proof. 1 is proven from the previous proposition. Next, I set out to prove 2. Since types are observable, the firm has two pricing tools to extract all surplus from agents. The firm can extract all this surplus using a combination of p and f. In particular, for any p that the monopolist chooses, since C is a function of p, f will just be set so that $C(p) = f$. This is precisely the entire amount of surplus that agents can extract. To prove 3, note that I am focusing on steady-state equilibrium. Combining the steady-state conditions from (5) gives 3. To prove 4, note that given the price f charged by the monopolist, it is clear that all agents will participate. The mass of agents who participate will be determined by the steady-state condition given by $\mu = \frac{i}{2\epsilon}$. What needs to proven now is whether all of the agents will find a match. Since agents are uniformly distributed, they are essential identical and have the same ϵ. Since the distribution of agents on both sides is uniform, the probability of finding a match is given by

$$P(\text{finding a match}) = \frac{2\epsilon}{1} = 2\epsilon$$

Since each period is identical and independent, the probability that an agent has yet to find a match by time T is

$$P(\text{not found a match by time } T) = \prod_{t=1}^{T} P(\text{not found a match}) = (1 - 2\epsilon)^T$$

As $T \to \infty$, we have that $P(\text{not found a match by time } T) \to 0$, since $2\epsilon \leq 1$.

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