Pricing and Incentives in Defined Contribution Retirement Systems*

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Abstract

This paper studies the effects of different pricing schemes and auction mechanisms on overall surplus levels in a privately managed, defined contribution retirement system. One such system is in Chile, where retirement savings are mandatory and taken as a fixed percentage out of monthly salary. In Chile, a small number of banks compete on the prices they charge for managing a retirement account through bidding in a sealed-bid first-price auction. The winner of the auction gets monopoly rights over new consumers entering the labor market for two years. As with many other retirement systems, Chilean authorities have puzzled over the lower-than-expected level of retirement savings throughout the years. In this paper, we examine this problem by considering the effects of different pricing schemes on firm behavior. In particular, we develop a model that captures the strategic interactions between firms as well as demand characteristics to derive the pricing and bidding schemes that maximize efficiency. We consider a repeated auction with effort and compare three different pricing schemes – the first is a fixed price that is paid regardless of the returns that a firm provides, the second is pricing based on returns whereby the firm can charge higher prices if they are able to get higher returns for their customers, and the third is a hybrid pricing scheme where firms bid on a guaranteed rate of return rather than taking a management fee as a fixed percentage of investment. We prove that the third pricing scheme is the most efficient.

JEL Classification: J26, J32, J38

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1 Introduction

Demographic changes (e.g., increased longevity, decreased fertility rates) have caused countries to move from either unfunded pay-as-you-go (PAYG) or defined-benefit (DB) systems, to fully funded defined-contribution (DC) systems or to multipillar (MP) approaches with individual DC accounts.[1] For instance, between 1980 and 2008, workers in the U.S. covered by DB systems decreased by 18%, while workers covered by DC systems increased by 23% (Butrica et.al. [2009]). In the American private sector, between 1975 and 2012, workers covered by DB plans decreased from 74% to 6% (Keim and Mitchell [2016]).

The architecture of DC systems varies across countries. An iconic model is the Chilean system. Chile was one of the first countries to implement a private retirement system in 1981 when it switched from a DB PAYG to a DC system. With support of the World Bank, the Chilean experience became an archetype for the implementation of defined contribution private retirement systems in Latin America, Europe, and Asia (Orszag and Stiglitz [2001]).[2]

In privatized DC systems, savings are usually managed by private companies known as pension fund administrators (PFAs). PFAs are responsible for investing individuals' savings, for whose service they charge a fee. Fee levels and structures vary across countries and are typically charged as a share of contributions, benefits, assets managed or as an annual fixed amount (Creighton and Piggott [2006]).

Due to the wide variation of DC systems around the world, its own market characteristics, competition levels, maturity, among others; one issue that is not clear is the effect of alternative pricing schemes on outcomes such as market equilibrium prices and firm's performance. As we lack of counterfactual evidence on running system, one question that remains unanswered is what is the effect of charging a fee over contributions (wages) or a fee over returns on firm's performance, and therefore on individual's accumulated wealth.

In this paper we investigate such question. We theoretically study the effect of different pricing schemes and bidding mechanisms for monopoly rights to new enrollees on equilibrium market prices in the PFAs market in the setting of DC private retirement system. We incorporate the main institutional features of the Chilean DC system and develop a model that captures the strategic interactions between firms as well as demand characteristics to evaluate the effect of alternative pricing and auction mechanisms on market surplus. To test the hypothesis that alternative pricing schemes impact firm's strategic decisions differently, we compare three pricing

[1] In unfunded systems, aggregate benefits are financed by current workers, while in funded systems benefits are financed by the returns on accumulated savings. In DC systems, the contribution rate is exogenous while benefits are endogenous. In DB retirement systems contribution rates are endogenously set in order to achieve a fixed lump-sum retirement income or a pension determined by individuals' previous earnings levels. In MP models, there is typically a safety net (first pillar), a privately or publicly managed funded pillar, and a voluntary private pillar.

[2] Examples of countries that have implemented this model are Peru (1993), Argentina (1994), Colombia (1994), Mexico (1997), Czech Republic (1994), Hungary(1998), Poland (1999), Bulgaria (2000), Estonia (2002), among others. Recently developed countries, such as the U.S., have also discussed the implementation of such a system (Joubert [2015], Joubert and Todd [2016]).

schemes for bidding on new enrollees: one where fees charged by PFAs do not depend on their performance (e.g., fee over income), and therefore, on the returns they generate on individuals savings, one where fees are a function of such returns (e.g., fee over returns), and finally a two-part tariff (e.g., fee over income plus a fee over returns). Additionally, we compare the current first-bid design with a second-price auction.

For achieving adequate pension levels, policy makers in DC systems care about incentives on two sides of the market – both i) accumulated assets and savings choices by individuals, and ii) the incentives of pension fund administrators to determine their prices and portfolio decisions. With respect to individual behavior, there is an important strand of the literature that analyzes individual choices and stickiness in behavior in DC systems. It has been evidenced for instance that workers tend to follow default behavior rather than actively making decision, both for participating in DC systems and for defining contribution rates (e.g., Madrian and Shea [2001], Thaler and Benartzi [2004], and Carroll et.al. [2009]).

With respect to the supply side, the effect of fees on the efficacy of a system have received limited attention in the literature (Creighton and Piggott [2006]). There is also a debate on whether DC systems should assign firms the role to actively manage investments (or not). As the pricing scheme depend on the role of the PFAs, this affects how the fee should be defined. Under the hypothesis that by actively managing investments returns are higher, then one could argue in favor it. Nevertheless, there is evidence that PFAs tend to herd, which may be driven by imitation, momentum trading, and for reducing risk (Raddatz and Schmukler [2013]). Different pricing schemes may generate different incentives to firms. For instance, fees over generated returns may increase the firm's effort and therefore its performance; while fees over wages, as do not depend on returns, may reduce firm's incentives to invest effort in actively managing retirement savings.

The managed mutual fund market may be one of the closest market to PFAs. In this setting, there is evidence that long run returns of index funds outperform actively managed mutual funds in the U.S. (Shah and Fernandes [2001]). On the other hand, there is also evidence that actively managed mutual funds charge lower fees when they face more competitive pressure from indexed funds (Cremers et.al. [2016]). In the retirement context, this may mean that as more PFAs are less active in investments, fees on firms that differentiate may decrease. As there is variation on the fee structures and firm incentives across systems, it is relevant to study whether alternative pricing schemes within a retirement system affect firms' incentives to perform well and to differentiate from each other.

Under Chile's pension system, employers in the formal sector must contribute 10% of their earnings to an individual savings account that is managed by one of the six PFAs in the market. PFAs charge a fee over individuals earnings. In 2016, the average fee in the market was 1.15%. To increase competition levels in the market, in 2008, a reform was introduced for allowing auctions of new enrollees. These auctions take place every 24 months and are sealed-bid, first price auctions for monopoly rights to all new enrollees (primary young incoming labor force). Since 2010, new enrollees are automatically assigned to the firm with the lowest bid and after 24 months they can freely chose whether to stay or switch to a different PFA. The market does not allows price discrimination among new and old enrollees, and therefore the winning bid must be extended to all enrolles affiliated to the winning auction firm.

Our model explicitly models the first-price auction used in the Chilean retirement system. First, we study the baseline case where fees are charged over the enrollees' income and where firm effort in attaining a high return is not modeled. We then characterize the Nash equilibria for the first-price auction and contrast it to an alternative design with a second-price auction in this environment. Then, we extend the model to one model where effort is explicitly modeled. Under this extension, we consider new pricing strategies – a) pricing based on returns with unobservable effort and b) a two-part tariff schedule, where firms charge a fixed component for a promised level of returns, and charges additional higher prices if it is able to invest effort to generate higher returns. We find that the two-part tariff strategy is the one that maximizes total economic surplus in the market. This result is useful for policy-makers who may not only care about the performance of firms in the market, but also the surplus captured by the consumers in terms of achieving high returns on their retirement savings. We also show that the current Chilean policy where the maximum bid in a subsequent auction must be lower than the winning bid in the previous auction causes market unraveling, with firms exiting the market in finite time.

The paper is structured as follows. The next section presents the institutional background of the Chilean retirement system. The third section presents the literature review, the fourth presents the baseline model, the fifth presents extensions to the model, the sixth presents the welfare comparisons between all types of pricing, and the seventh section concludes.

2 Relevant Literature

This paper is an intersection of a few strands of literature – the first being the vast amount of research on Chilean and Mexican pension systems, the second being the study of incentives and risk aversion and how it relates to retirement fund choices, and the third being auctions. In the following section, we will summarize the most related papers in each of the strands.

There is well established evidence that contributory retirement systems have improved economic outcomes such as savings levels, labor force participation and productivity, and economic growth (Thomas and Spataro [2016]). This result is consistent with the evidence from the Chilean experience (see for instance, Schmidt-Hebbel [1998], Morandé [1998], Samwick [1998], Coronado [2002], Rezk et.al. [2009], Corbo [2003], James and Edwards [2005]).

Despite this advantages, it has been observed that the concentration in PFA's market occur in countries with open-non-employers fund have been implemented and this challenges competitive outcomes (Holzmann et.al. [2005]). For Chile, the price markup in the PFA's market has been documented (Arrau and Valdés [2002]). Difference policies have been discussed and implement for increasing competition. The proposals for introducing auctions were discussed by Valdés [2006] where auctions of groups of up to 500 thousand consumers per firm where suggested. The auctions mechanism rely on the idea that competition for monopoly rights should decrease costs and therefore market prices (Fisher et.al. [2006]).

In addition to the market structure, differences between prices and costs is also explained by the little price response from consumers. This phenomenon has explain by different factors. For instance, Mitchell et.al. [2007] find that financial illiteracy is a component of the market frictions in the fund management market. Hastings et.al. [2011] use a randomized experiment and find that individuals are more responsive to pricing information the relative benefits of different investment choices presented. Illanes [2016] finds that consumers over-value returns differences relative to price differences; consistent with the evidence from the Mexican security system. He finds that with no switching costs, prices may drop almost by 50 percent. This result goes in the same line than the ones from Luco [2016]. In particular for Mexico, Hastings [2010] allows for heterogeneity in financial knowledge, and finds that more educated and higher-income individuals place equal weight in management fees and past returns, while low-educated and low-income enrollees put more weight on fees. From the Mexican experience, there is also evidence that demand elasticity is affected by brand loyalty, the way fee information is presented to consumers, and by the firm's quality of service measured by the number of branch locations in an area or exposure to sales force (Hastings [2008], Hastings et.al. [2013]).

Since the auctions between the firms in Chile are conducted every two years, this paper is essentially one on repeated auctions. A commonly studied problems in repeated auctions is one of collusion. (Aoyagi [2003]) studies bidder communication in repeated auctions with no side transfers. They found that in infinitely repeated auctions, collusion is possible through intertemporal payoff transfer even if there is no side payment of money. In other words, bidders in repeated auctions can collude through the adjustment of continuation payoffs in a way that partially compensates for the lack of monetary transfer. This is similar to (Skrzypacz and Hopenhay [2004]), in which it was shown that bidders can still collude even without explicit communication and limited monitoring in the context of cartels. In this paper, we sidestep the issue of collusion since 1) the Chilean retirement market is heavily monitored and regulated by the government, and 2) there have been fewer than 10 auctions conducted in the history of the market. In particular, it does seem that side payments and transfers are not possible. This would mean that the popular result from (McAfee and McMillan [1992]), in which the authors show that the ability of bidders to collude depends on the ability to make side payments, does not hold in our paper.

3 Chile's Retirement System: Institutional Background

3.1 Participants, Market Shares, and Prices

Participants in the Chilean retirement system can be classified are either enrollees, pension fund administrators, or the government. The role of the government is to supervise the pension fund administrators market through a special division called the Superintendence of Pensions (SP).

Enrollment to the system is mandatory and automatic for employees. Self-employed workers and non-employed individuals may voluntary enroll.[3] Individuals are enrolled into one firm and every period, mandatory contributors are required to save a 10 percent of their wage deducted from their paycheck, up to a cap, in a savings account established for accumulate mandatory contributions. If the individual stops working, she is not forced to contribute but she must keep her account (therefore, it is likely that firms will have more enrollees than contributors). These savings can only be cashed upon retirement (minimum retirement age is 60 years old for women and 65 years old for men). Table 1 presents the market equilibrium quantities and shares.

The PFAs are for-profit private firms whose objective is to manage individuals savings and pensions. They charge a price over individual income (therefore know as variable fee as it varies across individuals).The fee is firm-specific as it does not depend on the investment fund (product) that the individual chooses. Fees to contributors are the main source of revenue for firms. From the reports of management costs published every semester by the Superintendence of Pensions (as of June 2016), fees represent more than 90 percent of total revenue.[4]

The pension fund administrators are free to define the fee within the legal framework: for increasing prices the firm must give a 90 days notice and for decreasing prices a 30 days notice. The variable fee has decreased over time together with the number of firms in the market (see Table A.1). Currently, it ranges between 0.41 and 1.54 percent, with a mean price of 1.15 percent. Starting in 2008, the average market fee have decreased substantially, influenced by the auction mechanism implementation in 2010 (see Figure 1). Non-winning auctions firms have tended to keep their prices while the important drop in fees are coming from the two winning firms (Figure 2). Prices per firm for the years 2010 to 2016 are presented in Table A.2.

[3]Voluntary participation has been historically low. After a reform was implemented in 2008 and a law passed in 2016, participation of independent workers is mandatory starting January of 2018. Gradual voluntary enrollment with an opt-out scheme for independent workers was implemented starting January of 2012. As of December 2016, mandatory contributors represented a 97 percent of market participation (see Table 1 for details).

[4]Before 2008, there was also a lump-sum fee (known was fixed fee). This fee was eliminated in 2008 as it was not an important component on firms revenue but had an important effect on individuals savings balances, specially for low-income individuals (see Castro [2005]).

Table 1: Market equilibrium quantities and shares

Firm	Active Enrollees	Contributors			
		Total	Mandatory	Voluntary	
				Independent	Non-employed
Market equilibrium quantities (number of individuals)					
Capital	1,712,740	910,142	883,881	26,158	103
Cuprum	622,563	428,743	418,964	9,614	165
Habitat	2,038,030	1,142,507	1,120,887	21,299	321
Modelo	1,501,765	684,090	657,370	26,269	451
Planvital	1,161,224	548,591	530,003	17,091	1,497
Provida	3,142,115	1,570,264	1,528,289	41,954	21
Total	10,178,437	5.284,337	5,139,394	142,385	2,558
Market equilibrium shares (%)					
Capital	16.83	17.22	17.20	18.37	4.03
Cuprum	6.12	8.11	8.15	6.75	6.45
Habitat	20.02	21.62	21.81	14.96	12.55
Modelo	14.75	12.95	12.79	18.45	17.63
Planvital	11.41	10.38	10.31	12.00	58.52
Provida	30.87	29.72	29.74	29.47	0.82
Total	100.00	100.00	100.00	100.00	100.00

Note: (a) Based on data by the Superintendence of Pensions of Chile. (b) Information at December 31 of 2016.

Figure 1: Evolution of market fee over wage (1988-2016)

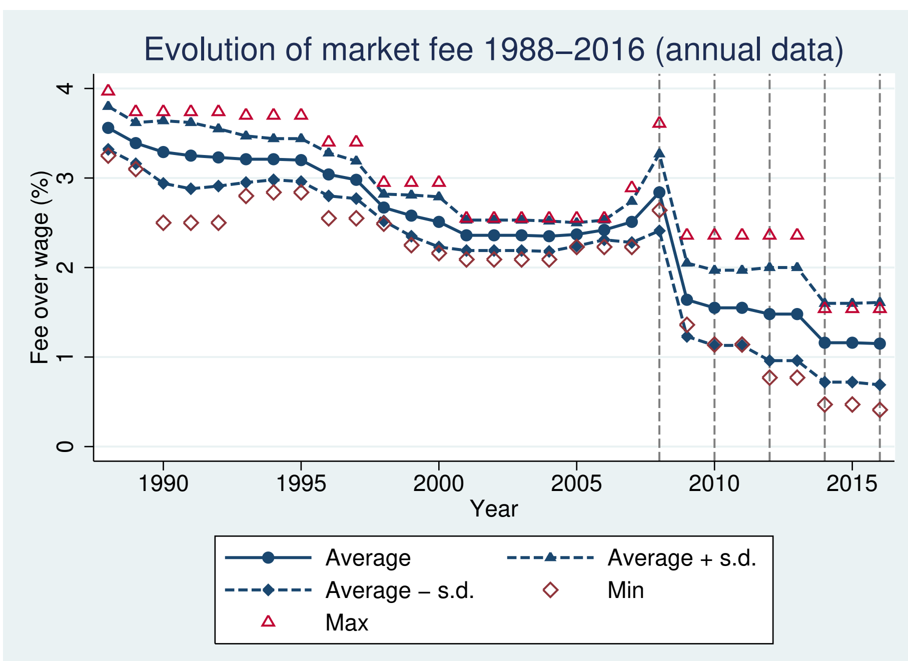
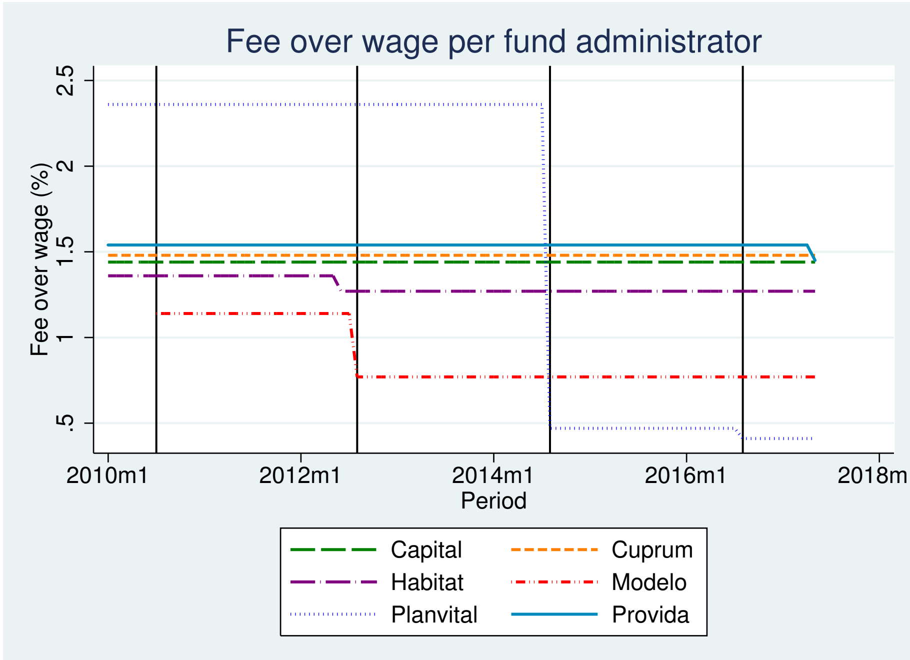


Figure 2: Fee over wage per firm (2010-2016)



3.2 Product Description

The PFAs consists capitalize retirement savings by investing them in financial instruments. Since 2002, the PFAs are required to offer at least 4 of 5 different investment accounts (known as multi-accounts) that vary in their level of financial risk. Firms can freely choose the share of investment on equities in each of these accounts, within an interval. The limits for investment on equities are the following: 80–40% for Account A, 60–25% for Account B, 40–15% for Account C, 20–5% for Account D, and up to 5% for Account E. Account A is the riskier account, and it is voluntary to offer, although it has always been available in all firms. Firms are required to report their investments to the SP. Every month, the SP publicly report each firm's portfolio.

Since the implementation of the multi-accounts, monthly returns have ranged within -22% to 10%, with standard deviations from 1% to 4% (see Table 2).

Firms differentiate on the returns that they are able to get on workers' investment. It is expected that firms that put more effort in managing investments should generate higher financial returns. Returns per year (annualized rate) range between 3 and 6 percent considering the entire period since the introduction of the multi-account system (see Table A.3 for 12-months, 36 months, and total returns on investment funds per account.)

Enrollees can chose up to 2 accounts for invest their savings. As of December 2016, around 11 percent of active enrollees invested in 2 accounts. They can switch accounts as many times as desired, although they might need to pay a fee if they switch more than twice in a calendar year. If an individual is enrolled in the system, but she is not a contributor any more, she can still make investment decisions. In Table 3 we present the market equilibrium quantities and shares per firm per product offered.

Table 2: Market average monthly return per account (%)

Fund	Share on equities	Oct 2002–Dec 2016	Aug 2010–Dec 2016
Account A	40–80%	0.59	0.28
		(3.51)	(2.82)
		[-22.44–10.23]	[-6.48–6.65]
Account B	25-60%	0.48	0.26
		(2.54)	(2.05)
		[-15.16–6.84]	[-4.21–4.44]
Account C	15–40%	0.42	0.28
		(1.71)	(1.38)
		[-8.84–4.38]	[-2.55–3.36]
Account D	5–20%	0.38	0.31
		(1.09)	(0.87)
		[-4.85–3.13]	[-2.49–3.02]
Account E	0–5%	0.32	0.32
		(0.87)	(0.85)
		[-3.17–3.84]	[-2.54–3.84]

Note: (a) Average monthly real returns per account. Standard deviation in round parentheses, minimum and maximum returns in square parentheses. (b) Own elaboration based on statistics from the Superintendencia de Pensiones.

Table 3: Market equilibrium shares per product per firm (%)

Firm	Account					
	A	B	C	D	E	2 accounts
Capital	12.25	25.76	36.58	15.68	9.73	13.01
Cuprum	20.02	19.03	32.52	10.40	18.03	11.51
Habitat	11.27	29.09	38.76	11.37	9.52	14.59
Modelo	4.88	70.39	13.69	4.69	6.34	3.07
Planvital	4.96	57.20	22.04	8.88	6.92	4.84
Provida	10.26	27.43	41.26	15.23	5.82	13.30
Total	10.10	36.07	33.55	12.08	8.20	10.92
# Enrollees	1,140,564	4,072,252	3,787,953	1,364,139	925,472	1,111,943

Note: (a) Constructed based on data by the Superintendence of Pensions of Chile. (b) Information at December 31 of 2016. (c) Share of individuals in 2 accounts is computed with respect to the number of active affiliated workers in each firm.

PFAs are responsible for providing a fund-specific annual real return for the last 36 months above a minimum level. This minimum level is defined by the lowest return computed between: (1) the mean annual real return for same fund considering the last 36 months for all the firms in the market minus 4 percent for Accounts A and B, and minus 2 percent for Accounts C, D and E; (2) the mean annual real return for same fund considering the last 36 months for all the firms in the market minus the (absolute value) of the 150 percent of such return. In case that the minimum level is not achieved, firms are require to compensate individuals. As show in Figure 5 such restrictions have not bind.

3.3 Cost Structure

The costs associated to the activities of pension fund managers are mostly distributed between labor and operating costs (see Table A.5). Some of these costs can be directly attributable to specific pension fund and savings balances managed (e.g., transaction costs in the financial markets) while other ones are associated to the firm level (e.g., operating costs attributable on running a sales office). In the model we will consider a variable provision cost c which captures the variable cost of providing the service per consumer. In Table A.6 we present the distribution of fund-specific-costs across funds.

Labor costs are associated to both management and sales workers. In 2016, 51 percent of total costs (21 percent sales labor costs and 30 percent management labor costs). These costs include for instance, commissions attributable to workers investing enrollees' retirement savings.[5]

Operating costs represent around a 40 percent of total costs and are distributed in management costs, which include for instance transaction costs in financial markets , technology, commercialization and other costs. Management operating costs represent around a 25 percent of total costs.

3.4 Auction Mechanism

After the reform of 2008, firms can participate in an auction for monopoly rights for 2 years to new enrollees. Starting in 2010, new affiliated workers have been assigned for 24 months to the PFA that offers to charge the lowest fee for the 24 months period to all individuals enrolleed in the PFA. An individual that was enrolled to the system before July of 2010 is not subject of being auctioned, although she can freely chose to enrolle in any firm. After 24 months, auctioned individuals can chose whether to stay or to transfer to another firm.

The objective of the introduction of auction was to increase competition in prices to effectively reduce market equilibrium prices. It was expected that this would have an effect on consumer's responses to prices and it would generate incentives to firms to differentiate in the quality (returns) of the products.

[5]An act passed by the Chilean Congress in 1997 restricts the amount of marketing activity that PFAs can run as it was found that sales and marketing costs where an important component of the PFAs balances.

The reform considers special circumstance under which new enrollees can transfer to another firm during the auctioned period. For instance, i) if the auction-winning-firm does not charge the lowest fee for 2 consecutive months as some other firm decreased prices, ii) if the return provided by the firm is under the minimum required return in the market, iii) if the lowest fee does not compensate the financial gain that the individual could have earn in a different firm.

The design is such that the winning bid must be below the current winning fee meaning that the winning fee must decrease over time. The auction can result deserted if this condition is not fulfilled and new enrollees would be assigned to the firm that offers the lowest fee at the time the enrollee enters the market. As mentioned, market fees have effectively decreased driven mostly by winning-auction firms. The main change in market shares has been due to the entrance of a new firm and due to the assignment of monopoly rights. Consumers turnover has increased between 2010 and 2016 from 3 to 6 percent (Table A.4).

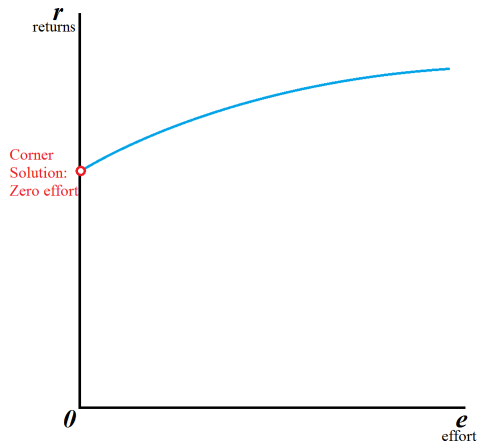
3.5 Stylized Facts for the Model

Based on the evidence presented in this section, we rely on the following stylized facts to model firm's interactions and demand characteristics.

1. Firms maintain prices for long periods of time, then drop to a very low price to win the auction.
2. Prices only change during auction periods, even amongst the firms that lose the auction.
3. Consumer demand is inelastic.
4. Winning bids must be decreasing over time – in particular, we show that more and more firms will drop out of the market as the number of auctions increases.
5. Firms don't raise prices.
6. Firms exhibit zero investing effort. This means that firm has the same portfolio (corner solution due to positive returns from zero effort. See Figure 3).

The baseline model implicitly or explicitly accounts for 1, 2, 3, and 6, but not 4 and 5. The section on model extensions account for 4 and 5.

Figure 3: Relationship between the returns on the investment product and the effort invested by the firm.



4 The Model

In this section, we present the baseline model in which we assume that all firms participate in a first price auction. We then analyze the effect of switching the auction mechanism to a second price auction scheme.

4.1 Baseline Model (First Price Auction)

Before we start the discussion of the model, we would like to highlight an intuitive but important consequence of the maximum bid policy in the Chilean system. This policy places an upper bound on subsequent winning prices from the auctions, which is inefficient since it is expected to induce all firms, over time, to leave the market. This is proven in the proposition below.

Proposition 1. *The maximum bid policy will cause all firms to exit the market in finite time.*

Proof. Fix c, the variable cost of providing the good. The equilibrium auction price is such that

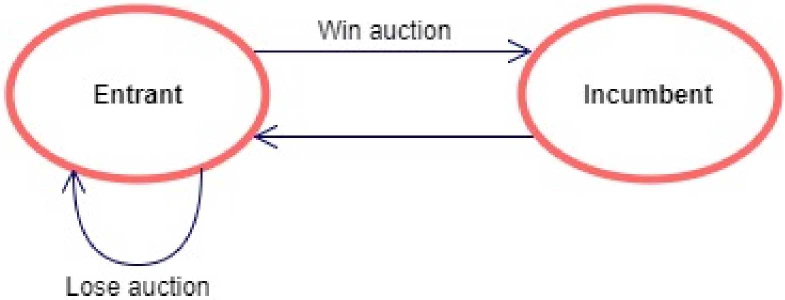
$$p_{t+1} < p_t - \epsilon \text{ for some } \epsilon > 0$$

for each time period t. Thus,

$$p_{t+1} < p_1 - t\,\epsilon < c$$

for $t > T$ for sufficiently large T. Thus, all firms will eventually be in a situation where the winning bid, and hence the price they can charge is below cost, causing them to exit the market. □

Figure 4: Possible States for Firms: Entrant and Incumbent



4.1.1 Pure Strategies

In this model, there are two sides – consumers and firms. There is a mass size 1 of consumers who live for three periods – young (y), middle-aged (m), and retired (r). When consumers are young, they are automatically assigned to the firm with the lowest bid in the auction for monopoly rights for the next period. When they are middle-aged, they have the option of switching to any other firm in the market but incur a switching cost s of doing so. For simplicity, assume that there are $\frac{1}{2}$ young consumers and $\frac{1}{2}$ of middle-aged consumers. Both consumers and firms discount at rate δ.

There are 3 firms who compete for consumers.[6] The firms participate in a sealed bid, first price auction for monopoly rights to all the young consumers in the subsequent period. Each firm faces the provision cost of c per consumer, where c is also the variable provision cost. In this first outline of the model, we will consider only pure strategies for the firm.

Since the firms compete Bertrand-style, the result is that firms will choose prices such that their expected utility over the lifetime of the consumers is zero. This being, there may be situations where firms price below marginal cost to generate surplus in the next period. To understand this, consider Figure 4. A firm can be in two states – it is either an entrant, in which case they have no consumers who are tied to them for the period, or they are an incumbent, which means they won the auction in last period and now have a mass of consumers who they have monopoly over for this period.

It is important to note that in this auction, as valuations increase, so does the bid. This is true by the fact that Bertrand competition forces firms to bid at their valuation. This being the case, it follows that as the bid increases, the probability of winning decreases. In this case, since we are consider pure strategies, the probability of winning is either 0 or 1. An incumbent will

[6]We construct the model using a finite number of firms to echo the Chilean retirement market, where there are 6 providers. Assuming that there are 3 firms in particular allows us to study competition between the two losers of the auction.

never remain an incumbent – since they have a mass of consumers who are tied to them, they have a higher valuation in the next round and so, will place a higher bid. This being said, an incumbent necessarily becomes an entrant in the subsequent round.

There are then only two prices that we must consider. The first is p_a, the lowest price offered in the auction, which will be a bid put forth by an entrant. The second price is p_I, the price offered by the incumbent from the previous period.

Given this setup, we can write out the Bellman equations for the firms. Since there are two states (Entrant and Incumbent), the two Bellman equations are given by the following value functions. We display the value function for the entrant first. The probability of winning the auction is $\frac{1}{2}$, as is the probability of losing the auction. In the case where the firm wins the auction, they will be able to collect $p_a - c$ from each of the $\frac{1}{2}$ consumers. In the case where they lose the auction, they proceed to next period as an entrant. The incumbent necessarily becomes an entrant in the next period. In this period, they are able to extract the amount $p_a + s - c$ from each of the $\frac{1}{2}$ consumers that are tied to it. This being, their value function is

Definition 1. *The value functions for the incumbent and entrants are given by*

$$V_E = \frac{1}{2}(\frac{1}{2}(p_a - c) + \delta V_I) + \frac{1}{2}\delta V_E \tag{1}$$

$$V_I = \frac{1}{2}((p_a + s - c) + \delta V_E) \tag{2}$$

Considering and solving the system of two equations given by 1 and 2 yields that

$$V_I = \frac{c - p_a - s + \frac{\delta s}{2}}{\delta(1+\delta) - 2} \tag{3}$$

$$V_E = \frac{p_a - c + \delta(p_a - c + \delta)}{2 - \delta(1+\delta)} \tag{4}$$

Since the firms are indifferent between winning and losing, we are able to solve for the optimal p_a from the indifference condition, given below:

Definition 2. *The indifference condition for the equilibrium in pure strategies is given by*

$$p_a - c + 2\delta V_I = \delta V_E \tag{5}$$

Thus, solving 3, 4, and 5 yields that

$$p_a = \frac{2c(1+\delta) - \delta(p_a + 2s)}{2+\delta} \text{ and } p_a = c - s\frac{\delta}{1+\delta} \tag{6}$$

Proposition 2. *The prices set by the incumbent and entrants in the pure-strategy Nash equilibrium are given by*

$$p_I = p_a + s = \frac{s}{1+\delta} + c \tag{7}$$

with the corresponding equilibrium value functions of

$$V_I > 0 \text{ and } V_E = 0 \ \forall \text{ parameter values} \tag{8}$$

The results from Proposition 2 can be interpreted very intuitively. The degree to which the incumbent can set high prices depends on the consumer switching cost, s. As s increases, consumers are less likely to switch away the incumbent after the two-year monopoly period and thus, the incumbent charges high prices to capture the surplus. The auction price then can be seen as the inverse of the incumbent price – the auction winner must price competitively with respect to the switching costs. If the switching costs are higher, the auction winner must price lower in order to induce the consumers to subscribe to their services. In equilibrium, the entrant is not able to capture any value in expectation due to the fact that they do not have any market power over any segment of consumers, unlike the incumbent, who enjoys monopoly rights for a period.

4.1.2 Mixed Strategies

Now, let's allow for the firms to use mixed strategies. Recall the baseline setup – there are three firms that are competing in a first-price auction. Firms compete in the auction over two segments of consumers – young and middle-aged.

Unlike the pure strategy setup where an incumbent in one round necessarily becomes an entrant in the next period, an incumbent in the mixed strategy equilibrium can still win in the next round but with a lesser likelihood. This is true since the bid distribution in equilibrium is unbounded – no matter how high a bidder places a bid, there is still a chance that the bid will win.

Let the bid strategy be p with a probability density function $f(.)$ and cumulative density function $F(.)$. In the following discussion, we will use F_E to be the cumulative density function for the entrant and F_I as the cumulative density function for the incumbent. Let p_L denote the minimum possible bid and p_W be the winning bid. Recall that the marginal cost of provision of services is c. There are then two cases–

1. $p_L \geq c$.

2. $p_L < c$.

For tractability, we restrict ourselves to the case of symmetric minimum bids, which allows us to disregard case 2. This is because if the winning bid was below cost, this must mean that the expected utility in that period is negative – thus, if any firm plays a bid such that $p < c$, they will make negative profits in that round and will leave the market.

For any auction in the mixed strategy equilibrium, there are two possibilities. The first is that the incumbent wins again with bid $p_W - s$, retains his young consumers from the last period (now middle-aged), and receives monopoly rights over the young consumers from this period. In this case, both of the entrants are left to complete Bertrand-style, and thus will both place bids equal to c.

The second possibility is that an entrant wins. Denote the winning entrant E_W and the loser as E_L. In this case, after the auction, E_W will play the bid p_W and E_L and the incumbent compete Bertrand-style for middle-aged customers. E_L and the incumbent will then be induced, by competition, to both bid at cost, playing c. However, since consumers have a switching cost s, the incumbent will win over E_L and keep its consumers from last period, who are now middle-aged.

Consider the two Bellman equations for the incumbent and entrant. Note that the Bellman equations are written assuming the bid placed is the minimum bid – hence the transition in the next period is necessarily to being an incumbent.

Definition 3. *The value functions for the entrants and incumbent are given by*

$$V_E = \frac{1}{2}(p_L - c + 2\delta V_I) \tag{9}$$

$$V_I = p_L - c + 2\delta V_I \tag{10}$$

Now, consider the indifference condition for solving for mixed strategies. The indifference condition must mean that all bids give an equally good payoff. Playing the minimum bid ensures winning, so the cumulative density function (CDF) of the minimum bid for the incumbent is $1 - (1 - F_E)^2$. The indifference condition is then

$$p_L - c + \delta V_I = (p_L - c + \delta V_I)(1 - F_E)^2 + (s + \delta V_E)(1 - (1 - F_E)^2) \tag{11}$$

Proposition 3. *In the mixed-strategy Nash equilibrium, the CDFs of the entrants and incumbent are given by*

$$F_E(p) = 1 - \frac{\sqrt{-2s + p_L(2+\delta) - c(2+\delta)}}{\sqrt{2p - 2s + p_L\delta - c(2+\delta)}} \tag{12}$$

$$F_I(p) = \frac{(-p\alpha - c(1+\delta)(-\alpha+\beta) + p_L(-\delta\alpha + \beta + \delta\beta))}{(-p + c - p_L\delta + c\delta)\sqrt{-2s + p_L(2+\delta) - c(2+\delta)}} \tag{13}$$

where

$$\alpha = \sqrt{-2(c+s) - c\delta + p_L(2+\delta)}$$
$$\beta = \sqrt{2p - 2s + p_L\delta - c(2+\delta)}$$

Proof. To solve for Equation 12, solve a system of three equations given by 9, 10, and 11. Then, let's solve the entrant's problem in order to solve for F_I. If the entrant loses, there are two cases. The first is that the other entrant wins. The second is that the incumbent wins again, which means that both of the entrants compete for middle-aged consumers. If the incumbent bids below c, the entrant will not compete. If the winning bid is above c, the entrant competes Bertrand with the other losing entrant and earns zero profit – this case is ignored. This give the entrant's indifference equation as

$$(\frac{p_L}{2} - \frac{c}{2} + \delta V_I) = (\frac{p}{2} - \frac{c}{2} + \delta V_I)(1 - F_I)(1 - F_E) + (F_E + F_I - F_E F_I)(\delta V_E) \tag{14}$$

Substituting the previous equations into equation 14 allows us to solve for the CDF for the incumbent.

□

4.2 Alternative Auction Mechanism: Second Price Auction

In this section we consider the implementation of an alternative auction mechanism. Let's assume that the firms participate in a second price auction instead of a first price auction. The reason why we consider a second price auction is because a first price auction allows for a multiplicity of equilibria, with the possibility of all firms charging high prices.

Generally, we find the same pure-strategy equilibrium since firms bid their valuations but we will eliminate all mixed strategy Nash equilibria. This is because all strategies of bidding above valuation are dominated by bidding exactly at valuation. There will, however, be more risk for collusion in the second price auction, as explained below, which makes the second price auction setup problematic.

Suppose that there the three firms have a valuation of 1, 2, and 3. In a non-collusive second price auction equilibrium, all bidders will bid their valuations and the winner who has a valuation of 1 will charge the second-lowest price, which is 2. In this situation, a collusive equilibrium could be the following: the firm with valuation 1 bids 1, while the other two firms bid 5, which allows for the possibility that firm 1 will charge 5 and make side payments to the higher-valuation firms to compensate them for their effort. This collusive equilibrium cannot be sustained in a first price auction since the winner charges their exact bid. Hence, the firms with valuations 2 and 3 cannot undercut firm 1 since this will be below their valuations.

Definition 4. *The value functions for the entrants and incumbent under a second-price auction mechanism is*

$$V_E = \frac{1}{4}(p_a - c + 2\delta V_I) + \frac{1}{2}\delta V_E \tag{15}$$

$$V_I = \frac{1}{2}(p_a + s) + \delta V_E \tag{16}$$

It is important to note that since the auction-winner gets monopoly rights to a segment of consumers for two periods, firms will typically price below marginal cost to generate rents tomorrow, hence implying that $p_a < c$. After the auction, the previous incumbent competes with the new incumbent, and not the other loser, who simply gives up, which gives the the value function of the incumbent above.

Solving Equations 15 and 16 gives that

$$V_E = -\frac{-c + p_a - c\delta + p_a\delta + s\delta}{2(\delta^2 + \delta - 2)} \tag{17}$$

$$V_I = -\frac{-2c + 2p_a + 2s - s\delta}{2(\delta^2 + \delta - 2)} \tag{18}$$

Proposition 4. *The equilibrium prices in a second-price auction are*

$$p_I = p_a + s = \frac{s}{1+\delta} + c$$

with the corresponding value functions

$$V_I = \frac{s}{2+2\delta} \tag{19}$$

$$V_E = 0 \tag{20}$$

Proof. Solving for p_a by using the indifference condition then gives that

$$p_a = \frac{c + c\delta - s\delta}{1+\delta}$$

Thus, the high (losing bid) is given by

$$p_I = p_a + s = \frac{s}{1+\delta} + c$$

Substituting the prices into the value functions for the entrant and incumbent then gives that

$$V_I = \frac{s}{2+2\delta}$$

and

$$V_E = 0$$

□

Thus, we arrive to the same conclusion as the model that uses a first-price auction mechanism, albeit with a higher likelihood of collusion, making second-price auctions inferior to first-price auctions. The same intuition from the first-price auction setup holds.

5 Extensions of Baseline Model

In this section we extend the baseline model to consider firm's effort. Based on this, we will consider alternative pricing schemes where the prices depend on firms' performance and retirement investment returns. The first two subsections will be dedicated to modeling firm's effort.

Let's suppose that the firm can choose a level of effort e to generate better returns on their customers' investments. We will look at two scenarios – the first where consumers can never observe how much effort each firms invests in their portfolios, and the second when consumers can.

5.1 Unobservable Effort

Each firm can invest effort e per consumer in their portfolios in each period. Consumers can never of observe this effort level but they do observe returns. Given e, consumers face an expected return of $r(e)$, where $r(e)$ is strictly increasing, positive, concave, and $r^{'}(0) > 1$. Bids are placed before effort is determined. Suppose that consumers know the set of possible firm strategies.

We use the following notation – when bidding, the bid level is p_a with a corresponding effort level e_a and the incumbent from the previous period has a bid p_I with a corresponding effort e_I.

Definition 5. *Given that p is the price that consumers pay on their portfolio, the indicator variable I_{switch} is 0 if there is no switching and 1 if consumer decides to switch, and s is the switching cost, the consumer utility in expectation is*

$$U = r(e) - p - I_{switch}s$$

The value functions for the entrant and incumbent are

$$V_E = \frac{1}{4}(p_a - e_a + 2\delta V_I) + \frac{1}{2}\delta V_E$$

$$V_I = \frac{1}{2}(p_I - e_I) + \delta V_E$$

with the indifference condition $\frac{(p_a - e_a + 2\delta V_I)}{2} = \delta V_E$.

Solving the two equations yields

$$V_I = \frac{-e_I(\delta - 2) + p_I(\delta - 2) + (e_a - p_a)\delta}{2(\delta^2 + \delta - 2)}$$

$$V_E = \frac{e_a - p_a + (e_I - p_I)\delta}{2(\delta^2 + \delta - 2)}$$

Solving for p_a from the indifference condition then yields that

$$p_a = \frac{2e_a + (2e_I + p_a - 2p_I)\delta}{2 + \delta} \tag{21}$$

Solving for p_a in 21, we have that

$$p_a = e_a - e_I\delta - p_I\delta \tag{22}$$

Proposition 5. *The equilibrium level of effort in the model with unobservable effort is*

$$e_I^* = 0 \text{ and } e_E^* = 0$$

Proof. Effort is only observed by consumer after a firm wins the auction – hence, it is strictly dominant to exert less effort. Thus, effort will be zero when you win the auction, as well as for the incumbent. It's helpful to note that this outcome is similar to that without effort since we are at a corner solution.

□

Now, consider the firm profit function, given by

$$(p - e)q$$

where p is the price that the firm charges, e is the effort level it exerts, and q is the mass of consumers that it serves. Since effort is unobservable, e is independent of q. Thus, we can rewrite the profit function as

$$p\,q - e$$

This being, the profit function is clearly maximized when effort is minimized. Denote the lowest amount of effort as e_0, where e_0 could be positive, representing that even the most inexpensive provision of this good has a marginal cost.

Proposition 6. *The equilibrium prices for the winner of the auction (p_a^*) and the incumbent (p_I^*) are given by*

$$p_a^* = c + \frac{\delta s}{1-\delta}$$

$$p_I^* = p_a^* + s = c + \frac{s}{1-\delta}$$

Proof. Substituting in e_0 into the Equation 22 gives

$$p_a = e_0 - e_0\delta - p_I\delta = -(1+\delta)e_0 - \delta p_I$$

Recalling from the baseline model with pure strategies that $p_I = p_a + s$, substituting into the equation for p_a gives

$$p_a = c - \frac{\delta}{1-\delta}s$$

Thus, if e_0 represents the firm's marginal cost, the unobservable effort case reduces to the baseline model with no effort. Note that the equilibrium outcome is the same for unobservable effort as with pure and mixed strategies with a first-price auction.

□

5.2 Observable Effort

In this extension, we consider the same setup as the previous section but now consumers observe how much effort firms invest in their portfolios. To find the indifference conditions, we consider the outside options for the consumers.

The dynamics of the auction are identical to the case with unobservable effort because consumers are forced to subscribe to the lowest-priced firm even if the firm has exerted minimal effort – that is to say, the effort level of the winning firm will always be zero. Thus, we only need to consider the problem of the incumbent firm. To avoid losing consumers to the auction winner, the incumbent must offer a higher expected utility given the following expression

$$r(e_0) - p_a - s \leq r(e_I) - p_I$$

where the right-hand side is the utility level offered by the incumbent and the left-hand side denotes the utility level offered by the winner of the auction.

Definition 6. *Consumers are indifferent when* $p_I = p_a + s - r(e_0) + r(e_I)$.

Given that firm profit per consumer served is $p_I - e_I$, substituting in for p_I gives that firm profit is $p_a + s - r(e_0) + r(e_I) - e_I$. Solving the simple optimization problem of choosing e_I yields the following proposition.

Proposition 7. *The efficient amount of effort exerted by the incumbent,* e_I^**, is*

$$r^{'}(e_I^*) = 1$$

Proof. Trivial and left to the reader. □

Thus, $r(e_I^*)$ and e_I^* are uniquely determined by firms' production technology and not competition. The price is then $p_I = p_a + s - r(e_0) + r(e_I^*)$. Recalling that $p_a = e_a - e_I^*\delta - p_I\delta$ and $e_a^* = e_0$, we have

$$p_a = e_0 + e_I^*\delta - (p_a + s - r(e_0) + r(e_I^*))\delta$$

Proposition 8. *In the model with observable effort in a first-price auction, the first-best level of effort is achieved by the incumbent and the entrants exert the minimum level of effort. In particular, prices set in equilibrium are*

$$p_a^* = \frac{e_0 + (e_I^* - s + r(e_0) - r(e_I^*)s)\delta}{1+\delta}$$

$$p_I^* = \frac{e_0 + s - r(e_0) + r(e_I^*)s + e_I\delta}{1+\delta}$$

with the corresponding value functions

$$V_E = 0$$

$$V_I = \frac{e_0 - e_I^* + s - r(e_0) + r(e_I^*)s}{2+2\delta}$$

Proof. Solving for p_a gives

$$p_a = \frac{e_0 + (e_I^* - s + r(e_0) - r(e_I^*))\delta}{1+\delta}$$

Substituting in $e_a^* = e_0, p_I = \frac{e_0+s-r(e_0)+r(e_I^*)s+e_I\delta}{1+\delta}, p_a = \frac{e_0+(e_I^*-s+r(e_0)-r(e_I^*)s)\delta}{1+\delta}$ yields

$$V_E = 0$$

and

$$V_I = \frac{e_0 - e_I^* + s - r(e_0) + r(e_I^*)s}{2+2\delta}$$

□

In conclusion, the first-best level of effort (e^*) is a positive amount while the minimum possible level of effort, e_0, is either 0 or a smaller amount than e_0^*, which gives us the following relationship

$$e^* > e_0$$

There is an inefficiency issue since the auction winner in any period wins the auction by virtue of price competition alone, and effort is exerted and observed only after the auction. To see this, notice that the auction winner will always exert an amount of effort that is less than the first-best level. The surplus generated by the firm who wins the auction is always greater under the first-best outcome

$$r(e^*) - e^* > r(e_0) - e_0$$

By taking prices into consideration, we note that since pricing is independent of effort level, profits will be higher under an effort level of e_0 instead of e^*.

Since consumers buy from the auction winner when they are young and stay with them when they are middle-aged, their surplus level for the firm that won the auction is

$$r(e_0) - e_0 + \delta(r(e^*) + e^*)$$

and the maximum surplus that could be generated over the lifetime of consumers is

$$(1 + \delta)(r^* + e^*)$$

This being said, the amount of inefficiency in this model is captured by the following equation

$$(1 + \delta)(r^* + e^*) > r(e_0) - e_0 + \delta(r(e^*) + e^*)$$

For now, we shall only consider the model with unobservable effort as it is a better representation of reality compared to the assumption of (perfectly) observable effort.

5.3 Two-Part Tariff Pricing With Unobservable Effort

Now we consider an alternative pricing scheme where consumers are charged a fee over wages and a fee over returns. Suppose that in each period, firms can choose to invest effort e in each portfolio. Given e, the expected return is $r(e)$, where $r(e)$ is increasing and concave, with $r(e_0) - e_0 > \epsilon$ for some $\epsilon > 0$, where e_0 is the minimum level of effort that is possible.

The auction is held before effort is determined. Suppose that consumers know firm strategies, but we restrict ourselves to non-reputational equilibria where consumers expect firms to only put in the effort that is individually optimal without reputation effects, and therefore firms do not invest effort to maintain reputation. Suppose also that there are two parts to the pricing – an initial price f per dollar invested and a return-share p.

Let's first solve the problem for the incumbent. The incumbents wins the auction with prices f_I and p_I and effort level e_I if the consumer gains from buying from it than a competitor with prices f, p and effort e,

$$r(e_I)(1-p_I) - f_I \geq r(e)(1-p) - f \tag{23}$$

Define $u_I = r(e)(1-p) - f$, the minimum level of utility that the incumbent must provide in order to win the auction. Thus, given p_I, Equation 23 gives that $f_I = r(e_I)(1-p_I) + u_I$. Consequently, we need only determine p_I and the value for f_I will be determined correspondingly.

The incumbent then chooses p_I to maximize profit, which is given by

$$\Pi_I = r(e_I)p_I + f_I - e_I \tag{24}$$

Equation 24 can be condensed by substituting in for the values of f_I:

$$\Pi_I = r(e_I)p_I + r(e_I)(1-p_I) + u_I - e_I = r(e_I) - e_I + u_I$$

Since u_I is independent of the incumbent's optimizing decision, it will choose e_I^* such that

$$r^{'}(e_I^*) = 1 \implies p_I^* = 1$$

Thus, under this pricing scheme, incumbents also put in a first-best amount of effort.

Similarly, let's use this framework for determining the prices that win the auction and the corresponding effort levels. A firm wins the auction if

$$f(e_a)(1-p_a) - f_a \geq f(e)(1-p) - f$$

Define $u_a = f(e)(1-p) - f$. Then, given by p_a, who f_a must be $f(e_a)(1-p_a) + u_a$ Given that a firm wins the auction, the expected utility for future periods is unconditional on prices. Thus, maximizing period utility is sufficient. The firm will choose p_a to maximize

$$\Pi_a = f(e_a)p_a + f_a - e_a = f(e_a)p_a + f(e_a)(1-p_a) + u_a - e_a = f(e_a) - e_a + u_a$$

Which, as the case for the incumbent, gives that $p_a^* = 1$ with a corresponding first-best effort level of e_a^*. Given e_a^*, e_I^*, p_a^*, and p_I^*, we have that $u_a^* = -f_a^*$ and $u_I^* = -f_I^*$.

The intuition for this result follows. If we allow the firm to change a per-return price, it will choose to keep every dollar of returns it gains for itself, giving the consumer nothing. To compensate the consumers for this, the firm will provide a negative pricing for each dollar invested, which is effectively a provision of a fixed return over the amount of the investments that the consumer has made.

Definition 7. *The value functions for the entrant and incumbent in this model are given by*

$$V_E = \frac{1}{4}(r(e) - f_a - e + 2\delta V_I) + \frac{1}{2}\delta V_E \tag{25}$$

$$V_I = \frac{1}{2}(r(e) - f_I - e) + \delta V_E \tag{26}$$

The indifference condition is given by

$$\frac{r(e) - f_a - e + 2\delta V_I}{2} = \delta V_E \tag{27}$$

Solving equations 25 and 26 yields

$$V_I = \frac{2e + 2f_I + f_a\delta - f_I\delta - 2r(e)}{2(\delta^2 + \delta - 2)}$$
$$V_E = \frac{e + f_a + e\delta + f_I\delta - (1+\delta)r(e)}{2(\delta^2 + \delta - 2)}$$

Solving the system of three equations yields that

$$f_a = \frac{(f_a - 2f_I)\delta - 2e(1+\delta) + 2(1+\delta)r(e)}{2+\delta} \implies f_a^* = -\delta f_I - e(1+\delta) + (1+\delta)r(e)$$

Given that you are competing with a new incumbent, the outside option is $f_a - s$, and old incumbent utility is f_I. Thus, the consumer is indifferent when

$$f_a - s = f_I \implies f_I = f_a - s$$

This gives that

$$f_a^* = r(e) - e + \frac{s\delta}{1+\delta}$$

and

$$f_I^* = r(e) - e + \frac{s}{1+\delta}$$

In summary, we have the following proposition:

Proposition 9. *The first-best level of effort by both the incumbent (e_I^*) and auction winner (e_a^*) can be achieved under this two-part tariff scheme with the corresponding equilibrium prices $p_a^* = p_I^* = 1$ and $f_a^* = r(e_a^*) - e_a^* + \frac{s\delta}{1+\delta}$ and $f_I^* = r(e_I^*) - e_I^* + \frac{s}{1+\delta}$.*

5.4 Pricing Based on Returns With Unobservable Effort

Now, consider a similar setup from the previous subsection but assume that pricing is based on returns. This is, firms set a price p that is a percentage of the returns that it gets to keep for itself. Denote p_a as the price set through the auction and p_I as the price set by the incumbent. The value functions for the entrants and incumbent are then. Firms can invest effort e per customer in their portfolios in each period. Given e, the generated return is $r(e)$, where the function r behaves as above.

Definition 8. *The value functions and indifference condition are:*

$$V_E = \frac{1}{4}(p_a(r(e_a) - e_a + 2\delta V_I) + \frac{1}{2}\delta V_E$$
$$V_I = \frac{1}{2}(p_I(r(e_I) - e_I) + \delta V_E$$
$$\textit{Indifference Condition: } \frac{(p_a r(e_a) - e_a + 2\delta V_I)}{2} = \delta V_E$$

Substituting the value functions into the equation for p_a, we have

$$p_a = \frac{p_a \delta r(e_a) + 2(e_a + e_I \delta - p_I \delta r(e_I))}{(2+\delta) r(e_a)}$$

Thus, the optimal value of p_a can only be solved given an explicit form for the function r.This being said, even without an explicit form for r, we can still draw some conclusions about the optimal range for p_a. In particular, we have the following proposition.

Proposition 10. *The effort level induced under this pricing scheme is inefficient.*

Proof. Suppose $p_a \geq 1$. Then a competitor can bid slightly below p_a with $p = 1 - \psi$ for a small $\psi > 0$ and win the auction with certainty. For a zero level of effort, we have $r(e_0) - e_0 > \epsilon$. Thus, for $\psi < \epsilon$, the competitor can make profit by undercutting, implying that this cannot be an equilibrium. Thus, p_a is necessarily less than 1, which means that the effort level must be below that which is optimal. □

5.5 Switching Costs that are Variable in Prices

Finally, consider a behavioral modification. Suppose that switching cost is s_H if the current firm has not raised prices, and s_L if it has. This takes into account the situation where consumers only pay attention to price increases and are strongly induced to change firms if prices increase.

People only switch after they have been with the incumbent. Incumbent prices are typically low and then they raise prices, so the relevant switching cost is s_L. Thus, the equilibrium outcome is just the above model with s_L in place of s. In practice, prices are falling and former incumbents do not raise prices, but keep them constant. s_L is smaller than decrement in price in each auction. This equilibrium is rationalized by having mixed strategies since the firm is moving from a high bid strategy to a lower one.

6 Discussion of Results

In the following table, we compare the overall total surplus generated by each modeling specifications that were considered in this paper. Recall that e_0^* is the minimum level of effort exerted in equilibrium, e^* is the first-best level of effort, p_a^* is the equilibrium price set in the auction, and p_I^* is the equilibrium price set by the incumbent.

Give the information in the table, it is clear that the extension to the baseline model with a first-price auction mechanism with unobservable effort and a two-part tariff pricing scheme (i.e. pricing over both returns and income) implements the first-best level of effort. t's important to note that the two-part tariff pricing scheme is not the unique pricing scheme that implements the first-best level of effort – in fact, there are many. The main takeaway is that pricing over both returns and incomes aligns the incentives of the firm in the retirement market with the incentives of the consumer who wishes for as high returns on her or his investment as possible.

Table 4: Summary of results

Model Type	Auction Type	Effort	Pricing Scheme	Total Surplus
Baseline	First Price	Not Modeled	Over Income	$(1+\delta)(r(e_0^*)-e_0^*)$
Baseline	Second Price	Not Modeled	Over Income	$(1+\delta)(r(e_0^*)-e_0^*)$
Extension	First Price	Observable	Over Income	$(r(e_0^*)-e_0^*)+\delta(r(e^*)-e^*)$
Extension	First Price	Unobservable	Over Returns and Income	$(1+\delta)(r(e^*)-e^*)$
Extension	First Price	Unobservable	Over Returns	$(r(e(p_a^*))-e(p_a^*))+\delta(r(e(p_I^*))-e(p_I^*))$
Extension	First Price	Unobservable	Over Income	$(1+\delta)(r(e_0^*)-e_0^*)$
Extension	First Price	Unobservable	Vary With Switching Costs	$(1+\delta)(r(e_0^*)-e_0^*)$

7 Conclusion

This paper studies the private, defined contribution retirement system in Chile by explicitly modeling a repeated auction with effort in order to derive the surplus-maximizing, and hence efficiency-maximizing, pricing scheme. We first characterize the Nash equilibria for the first price auction with no effort in this environment and compare the equilibria to that of a second price auction. We find that the second price auction allows for collusion and so, is inferior to the first price auction.

We then study different pricing schemes by explicitly modeling effort. We show that if effort was unobservable, firms will invest no effort in obtaining high returns for their portfolios. With observable effort, we consider three pricing schemes that can be characterized by their degree of variable pricing relative to fixed pricing – a scheme that involves only charging a fixed price (as is currently practice in the market), a scheme that involves only charging a price that varies with returns, and a two-part tariff scheme that involves a combination of the two previous schemes. We find that the two-part tariff scheme maximizes total economic surplus – that is, efficiency is maximized when firms compete in the auction on the fixed fee component, and then are incentivized to invest more effort in their portfolios in order to capture the variable fee component.

Appendix

A Evidence on the Pension Fund Managers Market

Table A.1: Historical average market equilibrium prices

Year	Variable fee (%)				Fix fee (dollars of 2016)				# Firms
	Mean	St.Dev.	Min	Max	Mean	St.Dev.	Min	Max	
1988	3.56	0.24	3.25	3.97	0.36	0.17	0.15	0.74	13
1989	3.39	0.23	3.10	3.74	0.31	0.19	0.00	0.74	13
1990	3.29	0.35	2.50	3.74	0.26	0.22	0.00	0.74	14
1991	3.25	0.37	2.50	3.74	0.26	0.22	0.00	0.74	13
1992	3.23	0.32	2.50	3.74	0.18	0.22	0.00	0.74	19
1993	3.21	0.26	2.80	3.70	0.14	0.17	0.00	0.43	21
1994	3.21	0.23	2.84	3.70	0.14	0.17	0.00	0.43	21
1995	3.20	0.24	2.84	3.70	0.14	0.18	0.00	0.43	16
1996	3.04	0.24	2.55	3.40	0.29	0.61	0.00	2.24	13
1997	2.98	0.21	2.55	3.40	0.32	0.62	0.00	2.24	13
1998	2.67	0.15	2.49	2.95	0.65	0.40	0.00	1.50	9
1999	2.58	0.23	2.25	2.95	0.69	0.41	0.00	1.50	8
2000	2.51	0.28	2.16	2.95	0.84	0.45	0.00	1.50	8
2001	2.36	0.17	2.09	2.55	0.91	0.48	0.00	1.50	7
2002	2.36	0.17	2.09	2.55	0.91	0.48	0.00	1.50	7
2003	2.36	0.17	2.09	2.55	0.91	0.48	0.00	1.50	7
2004	2.35	0.17	2.09	2.55	0.81	0.45	0.00	1.18	6
2005	2.37	0.13	2.23	2.55	0.70	0.42	0.00	1.04	6
2006	2.42	0.11	2.23	2.55	0.54	0.47	0.00	1.03	6
2007	2.51	0.23	2.23	2.89	0.36	0.44	0.00	1.03	6
2008	2.84	0.43	2.64	3.61	0.30	0.46	0.00	1.03	5
2009	1.64	0.41	1.36	2.36	–	–	–	–	5
2010	1.55	0.42	1.14	2.36	–	–	–	–	6
2011	1.55	0.42	1.14	2.36	–	–	–	–	6
2012	1.48	0.52	0.77	2.36	–	–	–	–	6
2013	1.48	0.52	0.77	2.36	–	–	–	–	6
2014	1.16	0.44	0.47	1.54	–	–	–	–	6
2015	1.16	0.44	0.47	1.54	–	–	–	–	6
2016	1.15	0.46	0.41	1.54	–	–	–	–	6

Note: (a) Based on data provided by the Superintendence of Pensions of Chile. (b) Information at december of each year. After September of 2008 the fix fee was eliminated. Fix fee for 2008 is measured at the month September which is the last period observed. (c) Fix fee expressed in dollars of December 30 of 2016.

Table A.2: Variable fee per firm at December of each year (% of wage) 2010-2016

Firm	Year						
	2010	2011	2012	2013	2014	2015	2016
Capital	1.44	1.44	1.44	1.44	1.44	1.44	1.44
Cuprum	1.48	1.48	1.48	1.48	1.48	1.48	1.48
Habitat	1.36	1.36	1.27	1.27	1.27	1.27	1.27
Modelo	1.14	1.14	0.77	0.77	0.77	0.77	0.77
Planvital	2.36	2.36	2.36	2.36	0.47	0.47	0.41
Provida	1.54	1.54	1.54	1.54	1.54	1.54	1.54

Note: (a) Based on data provided by the Superintendence of Pensions of Chile. (b) If no fee is reported, then firm is not in the market.

Table A.3: Real returns on investment Funds per firm (annualized return, %)

Firm	Account A			Account B		
	12 months	36 months	2002-2016	12 months	36 months	2002-2016
Capital	-1.10	3.69	6.08	0.94	3.74	5.24
Cuprum	-0.81	4.01	6.14	1.12	4.04	5.38
Habitat	-0.22	4.14	6.25	1.68	4.24	5.40
Modelo	0.26	3.89	–	2.14	3.90	–
Planvital	-1.62	3.10	5.73	-0.03	3.09	5.10
Provida	-1.59	3.51	6.04	0.35	3.60	4.97
Total	-0.87	3.85	6.13	1.07	3.88	5.23
Firm	Account C			Account D		
	12 months	36 months	2002-2016	12 months	36 months	2002-2016
Capital	1.63	4.02	4.67	2.69	3.91	4.50
Cuprum	1.95	4.53	5.19	2.82	4.45	4.80
Habitat	2.48	4.71	5.22	3.49	4.55	4.85
Modelo	2.76	4.03	–	3.76	4.13	–
Planvital	1.20	3.43	4.80	2.30	3.62	4.23
Provida	1.01	3.89	4.59	1.73	3.65	4.24
Total	1.74	4.24	4.90	2.60	4.07	4.54
Firm	Account E					
	12 months	36 months	2002-2016			
Capital	4.23	3.79	4.03			
Cuprum	3.95	3.85	3.98			
Habitat	4.32	4.00	4.13			
Modelo	4.43	3.55	–			
Planvital	3.55	3.18	3.29			
Provida	2.77	3.28	3.50			
Total	3.89	3.74	3.92			

Note: (a) Annualized rate of return. Constructed based on data by the Superintendence of Pensions of Chile. (b) Information at December 31 of 2016. (c) 2002-2016: September of 2002 to December of 2016.

Table A.4: Change in enrollees per firm, enrollees turnover and attraction (%)

Change in enrollees per firm (%)							
	Year						
	2010	2011	2012	2013	2014	2015	2016
Capital		-1.28	-2.06	-1.32	-2.35	-2.80	-2.50
Cuprum		1.27	1.48	1.12	1.21	0.65	-3.12
Habitat		-1.27	-0.95	-1.38	-1.97	-1.12	-0.69
Modelo		559.97	106.70	49.52	21.84	0.34	1.46
Planvital		0.31	0.53	-1.94	16.76	73.20	48.40
Provida		-2.31	-1.67	-2.62	-0.91	-1.54	-3.25
Total		2.36	3.48	2.76	2.32	2.21	2.18
Enrollees attracted per firm over total number of enrollees (%)							
	Year						
	2010	2011	2012	2013	2014	2015	2016
Capital	3.25	4.26	3.92	6.13	4.96	5.24	6.43
Cuprum	7.09	6.93	10.13	9.82	9.11	9.81	10.17
Habitat	2.15	2.14	2.39	2.71	2.51	4.13	5.01
Modelo	498.69	273.91	346.78	2.32	2.21	2.93	5.87
Planvital	6.51	8.25	8.88	10.15	8.27	4.84	5.05
Provida	2.30	1.96	2.45	3.95	4.34	4.50	4.58
Total	2.98	3.04	3.48	4.53	4.24	4.69	5.56
Turnover of enrollees per firm over total number of enrollees (%)							
	Year						
	2010	2011	2012	2013	2014	2015	2016
Capital	3.47	3.85	4.68	5.98	6.03	6.55	7.07
Cuprum	7.47	7.29	8.45	7.54	6.58	7.60	11.60
Habitat	1.99	2.14	2.33	3.27	3.49	3.65	4.22
Modelo	15.16	18.38	219.70	60.14	2.83	4.07	4.13
Planvital	5.54	5.73	6.63	10.00	5.91	3.18	1.98
Provida	2.51	3.01	3.16	4.22	3.71	4.34	6.12
Total	3.05	3.26	3.52	4.47	4.25	4.67	5.47

Note: (a) Based on data provided by the Superintendence of Pensions of Chile. (b)

Table A.5: Cost and Revenue Structure for AFPs (%)

	Year						
	2010	2011	2012	2013	2014	2015	2016
Revenue							
Fees	74.9	93.4	82.6	78.5	81.5	85.3	91.5
Reserve fund	14.1	-2.4	8.3	8.0	17.1	8.8	1.6
Other	11.0	8.9	9.1	13.6	1.5	5.9	6.9
Costs							
Labor costs	48.4	47.5	47.6	48.1	49.8	48.5	50.7
Management	28.2	27.9	29.0	29.0	30.6	28.6	29.5
Sales	20.2	19.6	18.7	19.1	19.2	29.9	21.2
Operating costs	37.2	40.0	41.5	36.6	40.2	42.1	39.8
Management	n.d.	25.6	25.8	24.4	22.1	24.7	24.5
Technology	n.d.	4.9	5.3	4.5	6.9	8.2	6.9
Commercialization	n.d.	3.2	4.7	4.2	5.8	4.2	3.6
Other	2.9	6.3	5.7	4.5	5.3	5.0	4.8
Depreciation and Amortization	11.6	11.4	11.2	11.5	8.9	9.2	9.5

Note: (a) Based on annual Reports of Management Costs from the Superintendence of Pensions of Chile. (b) Report for 2016 considers January -June 2016. (c) n.d. = no data

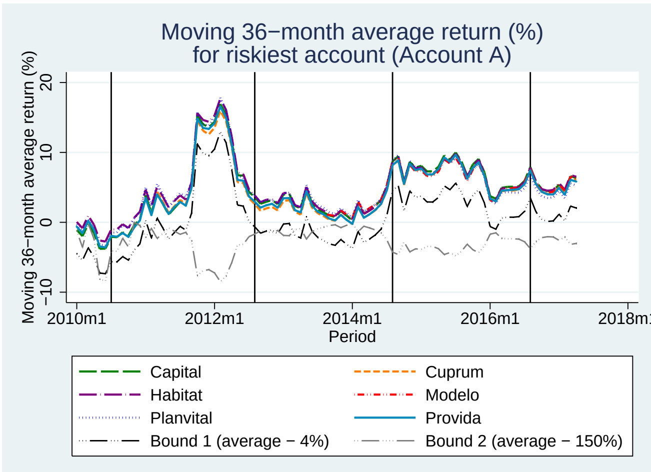
Table A.6: Distribution of Cost Attributable to Fund Management per Account (%)

	Year						
	2010	2011	2012	2013	2014	2015	2016
Account A	24.1	20.4	19.8	19.1	19.4	18.5	15.3
Account B	22.1	19.9	20.6	23.5	20.3	19.4	21.5
Account C	38.8	38.0	37.3	38.8	36.2	35.1	33.8
Account D	11.9	13.9	13.5	13.8	13.6	15.0	14.2
Account E	3.1	7.8	8.8	4.8	14.5	12.0	15.2
Attributable costs	47.3	46.0	35.5	21.1	31.9	29.6	90.0

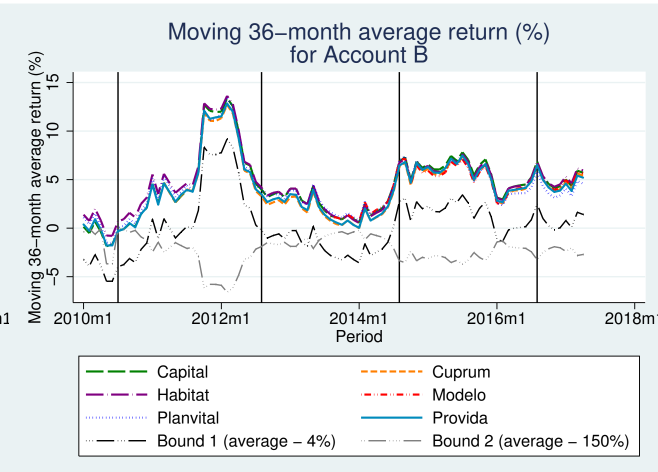
Note: (a) Based on annual Reports of Management Costs from the Superintendence of Pensions of Chile. (b) Report for 2016 up to June 2016. (c) Attributable costs = share of total costs attributable to the management of pension funds.

Figure 5: Return per account per firm (36-months moving average)

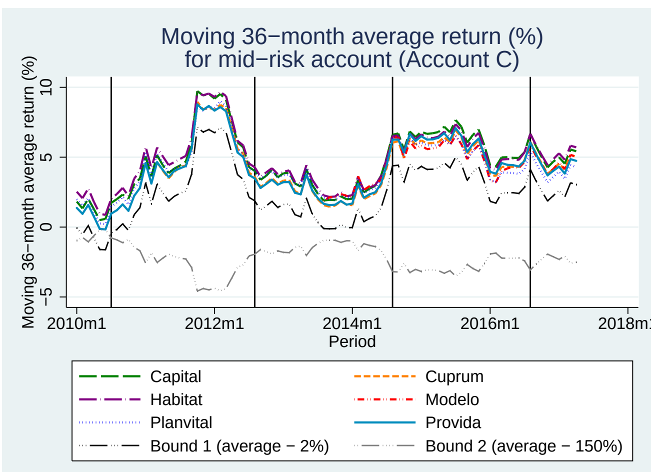
(a) Account A



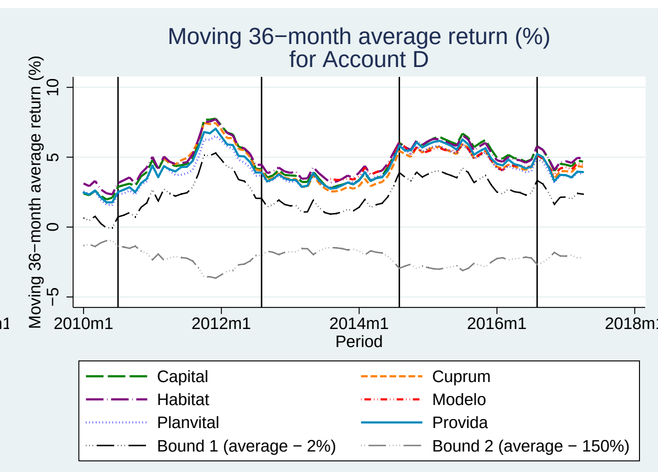
(b) Account B



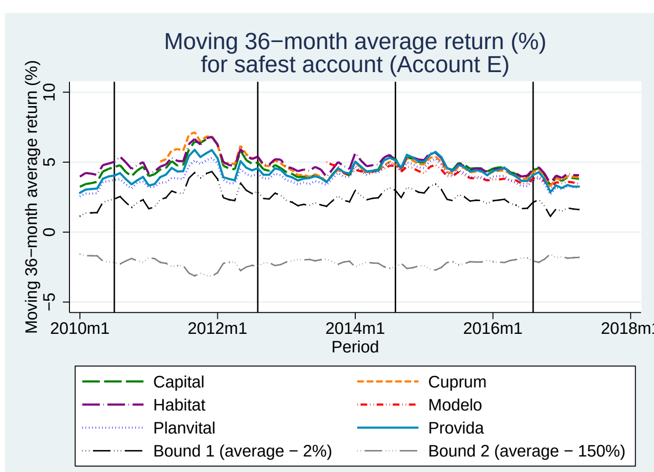
(c) Account C



(d) Account D



(e) Account E



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